Trinity College

WA Exams Practice Paper E, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

The points A, B and C have coordinates (4, 6), (10, -2) and (7, 10) respectively.

(a) Find the vector $\overrightarrow{BC}$. (1 mark)

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{OC} - \overrightarrow{OB} \\ &= \begin{bmatrix} 7 \\ 10 \end{bmatrix} - \begin{bmatrix} 10 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ 12 \end{bmatrix}\end{aligned}$$

(b) Find $|\overrightarrow{AB}|$ (2 marks)

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= \begin{bmatrix} 10 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 6 \\ -8 \end{bmatrix} \\ |\overrightarrow{AB}| &= 10\end{aligned}$$

(c) The point D divides the line segment CB internally in the ratio 2:3.

Find the position vector of the point D. (3 marks)

$$\overrightarrow{CB} = \begin{bmatrix} 3 \\ -12 \end{bmatrix}$$

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OC} + \frac{2}{5}\overrightarrow{CB} \\ &= \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \frac{2}{5}\begin{bmatrix} 3 \\ -12 \end{bmatrix} \\ &= \begin{bmatrix} 8.2 \\ 5.2 \end{bmatrix}\end{aligned}$$

See next page

Question 2 **(8 marks)**

A simple type of robot can be programmed to travel in a straight line with constant velocity.

Relative to an origin O, robot A leaves position $-13\mathbf{i}+22\mathbf{j}$ m and travels with velocity $3\mathbf{i}-2\mathbf{j}$ m/s.

One second later, robot B starts from position $5\mathbf{i}+15\mathbf{j}$ m and travels with velocity $-4\mathbf{i}-\mathbf{j}$ m/s.

(a) Calculate the position and velocity of robot A relative to robot B at the instant robot B starts and hence explain why the robots will not collide. (4 marks)

When B starts A is at $\begin{bmatrix}-13+3\\22-2\end{bmatrix}=\begin{bmatrix}-10\\20\end{bmatrix}$

$${}_A\mathbf{r}_B=\begin{bmatrix}-10\\20\end{bmatrix}-\begin{bmatrix}5\\15\end{bmatrix}=\begin{bmatrix}-15\\5\end{bmatrix}$$

$${}_A\mathbf{v}_B=\begin{bmatrix}3\\-2\end{bmatrix}-\begin{bmatrix}-4\\-1\end{bmatrix}=\begin{bmatrix}7\\-1\end{bmatrix}$$

No collision because ${}_A\mathbf{v}_B$ is clearly not a multiple of ${}_A\mathbf{r}_B$.

(b) Robot C, travelling with velocity $8\mathbf{i}-7\mathbf{j}$ m/s, leaves its initial position five seconds after A starts and collides with B, three seconds later. Determine the initial position of robot C. (4 marks)

At time of collision B is at $\begin{bmatrix}5\\15\end{bmatrix}+7\begin{bmatrix}-4\\-1\end{bmatrix}=\begin{bmatrix}-23\\8\end{bmatrix}$

Position of C $\begin{bmatrix}a\\b\end{bmatrix}+3\begin{bmatrix}8\\-7\end{bmatrix}=\begin{bmatrix}-23\\8\end{bmatrix}\Rightarrow\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}-47\\29\end{bmatrix}$

See next page

Question 3 **(6 marks)**

A true statement is 'if a hexagon is regular then it has six sides of equal length'.

(a) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

If a hexagon does not have six sides of equal length then it is not regular.

True – contrapositive statements are always true.

(b) Write the inverse of the statement and explain whether or not the inverse is also true. (2 marks)

If a hexagon is not regular then it does not have six sides of equal length.

False – sides can be equal so long as at least two of its angles are not the same.

(c) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

If a hexagon has six sides of equal length then it is regular.

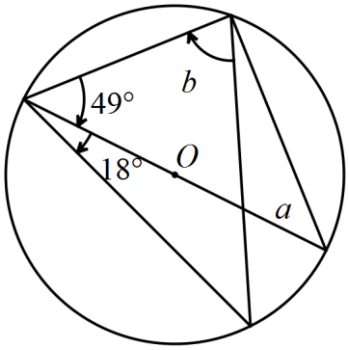
False – the angles must also be equal for a polygon to be regular.

See next page

Question 4 **(10 marks)**

(a) Determine the values of the pronumerals a, b and c in the diagrams below.

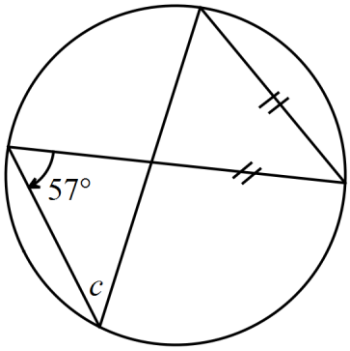
(i) (2 marks)



$a = 90 - 49$
$= 41°$

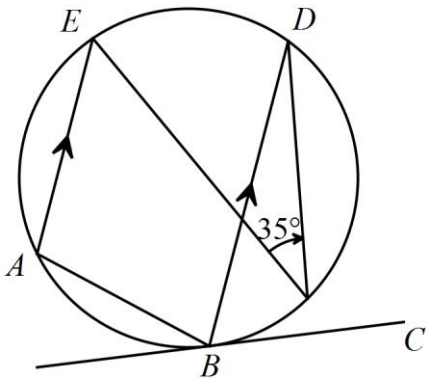
$b = 90 - 18$
$= 72°$

(ii) (2 marks)



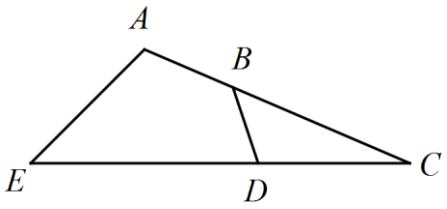
$c = 180 - 57 - 57$
$= 66°$

(b) Determine the size of angle ABC. (2 marks)



$\angle BEA = \angle EBD$ (alternate)
$= 35$ (angles on same chord)
$180 - \angle ABC = 35$ (alt seg theorem)
$\angle ABC = 145°$

(b) In the figure below, $AB = 2$, $BC = 4$, $CD = 3$ and $DE = 5$ cm. Prove that $ABDE$ is a cyclic quadrilateral. (4 marks)



$AC = 2 + 4 = 6$

$EC = 3 + 5 = 8$

$AC \times BC = 6 \times 4$

$= 24$

$= 8 \times 3$

$= EC \times CD$

Hence, by converse of intersecting chord theorem $ABDE$ is a cyclic quadrilateral.

See next page

Question 5 **(7 marks)**

A(2, 3), B(1, -2) and C(-3, 1) are the vertices of a triangle.

(a) State the vector $\overrightarrow{AC}$. (1 mark)

$$\overrightarrow{AC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

(b) Determine the exact value of $|\overrightarrow{BC}|$. (2 marks)

$$\overrightarrow{BC} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$|\overrightarrow{BC}| = \sqrt{4^2 + 3^2}$$
$$= 5$$

(c) Determine all vectors of magnitude 10 that are

(i) parallel to $\overrightarrow{BC}$. (2 marks)

$$\pm 2\begin{bmatrix} -4 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -8 \\ 6 \end{bmatrix} \text{ and } \begin{bmatrix} 8 \\ -6 \end{bmatrix}$$

(ii) perpendicular to $\overrightarrow{AC}$. (2 marks)

$$\left\| \begin{bmatrix} -5 \\ -2 \end{bmatrix} \right\| = \sqrt{29}$$

$$\pm \frac{10}{\sqrt{29}} \begin{bmatrix} 2 \\ -5 \end{bmatrix} \Rightarrow \frac{10}{\sqrt{29}} \begin{bmatrix} 2 \\ -5 \end{bmatrix} \text{ and } \frac{10}{\sqrt{29}} \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

See next page

Question 6 **(7 marks)**

(a) Use the method of contradiction to prove that a triangle with sides of 5 cm, 5 cm and 7 cm is not right angled. (4 marks)

Assume that the triangle is right angled, so that Pythagoras' Theorem can be applied and we can deduce that:

$5^2 + 5^2 = 7^2$

But

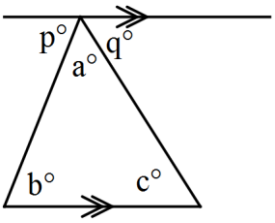
$5^2 + 5^2 = 25 + 25$

$= 50$

$\neq 7^2$

This result contradicts our original assumption and so the triangle cannot be right angled.

(b) Use the fact the angles on a straight line are supplementary to prove that the angle sum of a triangle is 180º. (3 marks)



The diagram shows a line drawn through the vertex of a triangle parallel to the base.

$\angle p + \angle a + \angle q = 180$ (given)

$\angle b = \angle p$ (alternate angles)

$\angle c = \angle q$ (alternate angles)

Hence $\angle b + \angle a + \angle c = 180°$ as required.

See next page

Question 7 **(8 marks)**

Two vectors are $\mathbf{c} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{d} = -12\mathbf{i} + 5\mathbf{j}$.

(a) Find

(i) $5\mathbf{c} + \mathbf{d}$ (1 mark)

$$5\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -12 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -15 \end{bmatrix}$$

(ii) $|\mathbf{d}|$ (1 mark)

13

(iii) $-|\mathbf{c}|\mathbf{d}$ (2 marks)

$$-5\begin{bmatrix} -12 \\ 5 \end{bmatrix} = \begin{bmatrix} 60 \\ -25 \end{bmatrix}$$

(b) Find $\mathbf{e}$ and $\mathbf{f}$ if $2\mathbf{e} + \mathbf{f} = 2\mathbf{c}$ and $\mathbf{e} - \mathbf{f} = \mathbf{d}$. (4 marks)

$$2\mathbf{e} + \mathbf{f} + \mathbf{e} - \mathbf{f} = 2\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} -12 \\ 5 \end{bmatrix}$$

$$3\mathbf{e} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -1 \end{bmatrix} - \mathbf{f} = \begin{bmatrix} -12 \\ 5 \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} 10 \\ -6 \end{bmatrix}$$

End of questions

Additional working space

Question number: _____

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